In-hour equation

Start: PRKEs. Assume: $C_i(t) = C_i e^{st}$ and $n(t) = ne^{st}$

Start with the delayed neutron precursor concentration equation.

$$\frac{\partial}{\partial t} C_i(t) = \frac{\beta_i}{\Lambda} n(t) - \lambda_i C_i(t) \quad i = 1...6$$

Assume $C_i(t) = C_i e^{st}$ and $n(t) = ne^{st}$

$$\frac{\partial}{\partial t} C_i e^{st} = \frac{\beta_i}{\Lambda} ne^{st} - \lambda_i C_i e^{st}$$

$$sC_i e^{st} = \frac{\beta_i}{\Lambda} ne^{st} - \lambda_i C_i e^{st}$$

Divide through by common terms.

$$sC_i = \frac{\beta_i}{\Lambda} n - \lambda_i C_i$$

$$C_i = \frac{\beta_i}{\Lambda(s + \lambda_i)} n$$

Now consider neutron concentration equation.

$$\frac{\partial}{\partial t} n(t) = \frac{\rho - \beta}{\Lambda} n(t) + \sum_{i=1}^{6} \lambda_i C_i(t)$$

Assume $C_i(t) = C_i e^{st}$ and $n(t) = ne^{st}$

$$\frac{\partial}{\partial t} ne^{st} = \frac{\rho - \beta}{\Lambda} ne^{st} + \sum_{i=1}^{6} \lambda_i C_i e^{st}$$

$$sne^{st} = \frac{\rho - \beta}{\Lambda} ne^{st} + \sum_{i=1}^{6} \lambda_i C_i e^{st}$$

Insert $C_i = \frac{\beta_i}{\Lambda(s+\lambda_i)} n$

$$sne^{st} = \frac{\rho - \beta}{\Lambda} ne^{st} + \sum_{i=1}^{6} \lambda_i \frac{\beta_i}{\Lambda(s + \lambda_i)} ne^{st}$$

Divide through by common terms.

$$s = \frac{\rho - \beta}{\Lambda} + \sum_{i=1}^{6} \lambda_i \frac{\beta_i}{\Lambda(s + \lambda_i)}$$

Multiply both sides by Λ. $\Lambda = \ell(1 - \rho)$

$$s\Lambda = \rho - \beta + \sum_{i=1}^{6} \lambda_i \frac{\beta_i}{(s + \lambda_i)}$$

$$s\ell(1-\rho) = \rho - \beta + \sum_{i=1}^{6} \lambda_i \frac{\beta_i}{(s+\lambda_i)}$$

$$\rho(s\ell+1) - s\ell = \beta - \sum_{i=1}^{6} \lambda_i \frac{\beta_i}{(s+\lambda_i)}$$

Bring β into summation.

$$\rho(s\ell+1) - s\ell = \sum_{i=1}^{6} \beta_i - \lambda_i \frac{\beta_i}{(s+\lambda_i)}$$

$$\rho(s\ell+1) - s\ell = \sum_{i=1}^{6} \beta_i \left(1 - \frac{\lambda_i}{(s+\lambda_i)}\right)$$

$$\rho(s\ell+1) - s\ell = \sum_{i=1}^{6} \beta_i \frac{s}{(s+\lambda_i)}$$

$$\rho = \frac{s\ell}{s\ell+1} + \frac{1}{s\ell+1} \sum_{i=1}^{6} \beta_i \frac{s}{(s+\lambda_i)}$$

Above is the in-hour equation. It is used to calculate the reactivity needed to put a nuclear reactor on a particular period.