Neutrons Slowing Down in Hydrogen without Absorption

Start: diffusion equation. Assume: Steady state, infinite medium, A=1, no absorption, and a source of neutrons $S(E)$ that exists from $E_* < E < E_0$.

Start with the diffusion equation.

$$\frac{1}{v}\frac{\partial \phi(E)}{\partial t} - \nabla \cdot D\nabla\phi(E) + \Sigma_t(E)\phi(E) = \int_0^\infty \Sigma_s(E' \to E)\phi(E')dE' + S$$

Assume steady state and remove the first term. Assume an infinite medium and remove the second term.

$$\Sigma_t(E)\phi(E) = \int_0^\infty \Sigma_s(E' \to E)\phi(E')dE' + S$$

Assume $\Sigma_s(E' \to E) = \frac{\Sigma_s(E')}{(1-\alpha)E'}$ for $E < E' < \frac{E}{\alpha}$. (Found from plotting energy vs. uniform probability of scattering between $\alpha\Sigma_s$ and Σ_s)

$$\Sigma_t(E)\phi(E) = \int_E^{E/\alpha} \frac{\Sigma_s(E')}{(1-\alpha)E'}\phi(E')dE' + S(E)$$

Assume slowing down in hydrogen without absorption. $\Sigma_t = \Sigma_s$ and $\alpha = \frac{A-1}{A+1}^2 = 0$

$$\Sigma_s(E)\phi(E) = \int_E^\infty \frac{\Sigma_s(E')}{E'}\phi(E')dE' + S(E)$$

Define $F(E) = \Sigma_s(E)\phi(E)$ take the derivative with respect to E.

$$\frac{\partial}{\partial E}F(E) = \frac{\partial}{\partial E}\int_E^\infty \frac{F(E')}{E'}dE' + \frac{\partial}{\partial E}S(E)$$

$$\frac{\partial}{\partial E}F(E) = -\frac{F(E)}{E} + \frac{\partial}{\partial E}S(E)$$

$$\frac{\partial F(E)}{\partial E}E + F(E) = \frac{\partial S(E)}{\partial E}E + S(E) - S(E)$$

$$\frac{\partial}{\partial E}F(E)E = \frac{\partial}{\partial E}S(E)E - S(E)$$

Integrate with respect to E' from E to E_0

$$\int_E^{E_0} \frac{\partial}{\partial E}F(E)E = \int_E^{E_0} \frac{\partial}{\partial E}S(E)E - \int_E^{E_0} S(E')$$

$$-F(E)E = -S(E)E - \int_E^{E_0} S(E')dE'$$

$$F(E) = S(E) + \frac{1}{E}\int_E^{E_0} S(E')dE'$$

Recall $F(E) = \Sigma_s(E)\phi(E)$ and $S(E) = 0$ for $E < E_*$

$$\phi(E) = \frac{1}{\Sigma_s(E)}\left[S(E) + \frac{1}{E}\int_{E_*}^{E_0} S(E')dE'\right]$$