Solution to PRKEs with one delayed neutron group

Start: PRKEs. Assume: one delayed neutron precursor group and $C_i(t) = C_i e^{st}$ and $n(t) = ne^{st}$.

Start with the neutron concentration equation.

$$\frac{\partial}{\partial t} n(t) = \frac{\rho - \beta}{\Lambda} n(t) + \lambda C(t)$$

Assume $n(t) = ne^{st}$ and $C(t) = Ce^{st}$

$$\frac{\partial}{\partial t} ne^{st} = \frac{\rho - \beta}{\Lambda} ne^{st} + \lambda Ce^{st}$$

$$sne^{st} = \frac{\rho - \beta}{\Lambda} ne^{st} + \lambda Ce^{st}$$

$$sn = \frac{\rho - \beta}{\Lambda} n + \lambda C$$

Apply same treatment to precursor concentration equation.

$$\frac{\partial}{\partial t} C(t) = \frac{\beta}{\Lambda} n(t) - \lambda C(t)$$

$$\frac{\partial}{\partial t} Ce^{st} = \frac{\beta}{\Lambda} ne^{st} - \lambda Ce^{st}$$

$$sCe^{st} = \frac{\beta}{\Lambda} ne^{st} - \lambda Ce^{st}$$

$$sC = \frac{\beta}{\Lambda} n - \lambda C$$

Using the sn and sC equations, solve for s, yielding solutions of the following form.

$$n(t) = n_1 e^{s_1 t} + n_2 e^{s_2 t}$$

$$C(t) = C_1 e^{s_1 t} + C_2 e^{s_2 t}$$

Assume $n(0) = n_0$ and $\frac{\partial}{\partial t} C(0) = 0$ to obtain an expression for C_0

$$\frac{\partial}{\partial t} C(t) = \frac{\beta}{\Lambda} n(t) - \lambda C(t)$$

$$0 = \frac{\beta}{\Lambda} n_0 - \lambda C_0$$

$$C_0 = \frac{\beta n_0}{\lambda \Lambda}$$

Solve for n_1, n_2, c_1, c_2 using the following four equations by inserting the general solutions into the PRKEs.

$$n(0) = n_0 = n_1 + n_2$$

$$C(0) = C_0 = C_1 + C_2$$

$$\frac{\partial}{\partial t} n(t) = \frac{\rho - \beta}{\Lambda} n(t) + \lambda C(t)$$

$$\frac{\partial}{\partial t} C(t) = \frac{\beta}{\Lambda} n(t) - \lambda C(t)$$